Value Added Tax: Do Enforcement Mechanisms Improve Compliance?

Aldo F. Ramirez Zamudio
Universidad de Lima

Abstract: Value Added Tax (VAT) is one of the main sources of internal revenues in Peru and most countries in Latin America.

Since the market in Peru shows a high heterogeneity in terms of firms' size and their economic transactions, it is not profitable to collect VAT from the smallest-sized firms or audit them. Therefore, there is a powerful incentive for evaders to create small-sized firms deliberately with the only aim of issuing invoices that do not support real transactions and selling them for a fraction of their nominal amount. Some big-sized firms "purchase" these invoices to overstate costs and reduce the VAT to be paid.

Peru's Tax Administration Service (STA, for its acronym in Spanish) is a public agency in charge of the collection and audits on internal revenues, and the Detailed Transactions Report (DTR) is a VAT's enforcement mechanism aimed to fight against this kind of VAT evasion.

The DTR is mandatory and the STA is able to impede commercial operations of any taxpayer that does not deliver it on time. However, since the number of small-sized firms in the market is large, but their average transactions are not considerable, the STA demands DTR only to big-sized firms.

The objective of this article is to prove that a VAT's enforcement mechanism, partially settled as in this case, becomes a perverse incentive for tax evaders to continue issuing and purchasing invoices that do not support real transactions.

We propose that requiring all taxpayers to submit DTR under the VAT's scope, will discourage this kind of evasion since reporting the DTR implies collusion's costs and a higher risk of being better controlled or audited.

Keywords: Value Added Tax (VAT) / tax avoidance / perverse incentive to evasion

INTRODUCTION

Value Added Tax (VAT) is an indirect tax that applies only to the added-value of a good or service and constitutes one of the most important sources of the Government's income in Peru and many other countries around the world. VAT law requires taxpayers to record detailed transactions in accounting books and support each transaction with a corresponding authorized invoice.

Das-Gupta and Gang (1996) consider that VAT has an important advantage over other sales taxes. They point out that the existence of purchase and sales invoices produces "self-enforcement", e.g., sellers would want to set a lower price than the sales price on the invoice so they may pay less tax, but buyers would want a higher price than the sales price so they may pay less tax.

Allingham and Sandmo (1972) assure that tax evasion may be seen as a gamble, they state that any rational individual is viewed as maximizing the expected utility of this gamble, weighting the benefits of successful cheating against the risky prospect of detection and punishment, and the individual pays taxes because he is afraid of getting caught and penalized if he does not report truthfully. This "portfolio" approach gives as a result that compliance depends upon audit and fine rates.

Alm (2012) affirms that Allingham and Sandmo (1972) focus exclusively on the financial incentives of the evasion gamble, and on the idea that individuals pay taxes only because they fear detection and punishment, but for many observers it is evident that compliance cannot be entirely explained by such purely financial considerations, especially those generated by the level of enforcement. Indeed, they insist taxpayers are also influenced by the social context in which, and the process by which, decisions are made; and they are motivated not simply by self-interests, but also by group notions like social norms, social customs, fairness, trust, reciprocity, tax morale, and even patriotism, as well as by individual notions of guilt, shame, morality, altruism, or alienation.

Franzoni (1999) defines compliance with tax law as a decision, meaning four elements: (i) accurate tax base reporting, (ii) correct liability calculation, (iii) timely filing of tax statements, and (iv) timely payment of the amounts due.

Mittone (2002) points out that VAT evasion usually involves three agents: the seller, the buyer and the Government. Interaction among these three agents may give rise to the following phenomena: the buyer of a

given good or service, may evade only if he is able to collude with the seller, who should behave as a tax collector for the Government. Collusion between the seller and the buyer would benefit both since they may reduce their burdens. The buyer would overstate costs, decreasing net payments, and the seller would not pay the VAT.

Galiani and Weinschelbaum (2011) find that many small-sized firms in Latin America operate informally because they are not productive enough to cover fixed cost. Indeed, informal economy in Peru is certainly colossal and its transactions while real are obviously made without the support of legal invoices. However, in this work we focus specifically on the problem caused when buyers overstate costs using invoices from sellers that do not pay the VAT; these invoices are legal, but transactions were never completed, and the corresponding tax was never paid. Most of the time these small-sized firms are created with the only purpose of issuing invoices not supported on real transactions[1]. This is by far the most common method to evade VAT in Peru and one of the most common practices used in South America.

Precisely, heterogeneity in the firms' size is an important issue in most developing countries. Figure 1 shows revenues by taxpayers' size in Peru. There is a notorious concentration of revenues in few taxpayers; more specifically, those denominated "Main Taxpayers" reach 13,706 or only 0.3% of the total number of taxpayers, but represent 82% of total revenues.

Usually, the biggest-sized firms are taxed on time, better controlled, regularly audited, and their remaining tax payments better enforced when necessary; however, it is more difficult and expensive to do the same for the large number of small-sized taxpayers.

There are 559,706 taxpayers under the VAT's scope in Peru. They sell and purchase goods and they may legally issue invoices which are used as credit for the VAT; however, most of them (546 thousand or 98%) are just medium and small-sized firms, barely controlled.

Since it is costly and difficult to enforce tax payments from small-sized firms, Peru's Tax Administration Service (STA) is continuously looking for lower-cost and more effective mechanisms. Hence, STA does not use costly seizures and audits for them, but other faster and lower-cost techniques. Unfortunately, the latter cannot legally determine major changes in their accounting (e.g. to prove that some transactions did not actually occur).

---

1   Once they are detected they just stop their operations and disappear.

Main Taxpayers

Mega — 41 Taxpayers: 22.0% of total revenues

Top — 208 Taxpayers: 13.2% of total revenues

Rest — 13 457 Taxpayers: 47.2% of total revenues

Medium and small sized Taxpayers

Vat general category — 383 Thousand Taxpayers: 17.2% of total revenues

Vat's special category — 163 Thousand Taxpayers: 0.2% of total revenues

Non Vat category small firms — 550 Thousand Taxpayers: 0.2% of total revenues

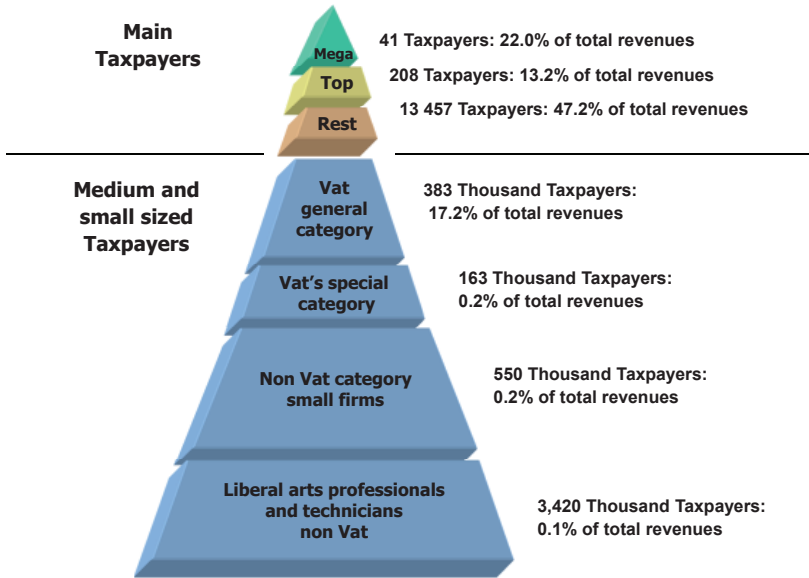Liberal arts professionals and technicians non Vat — 3,420 Thousand Taxpayers: 0.1% of total revenues

Figure 1: Tax Revenues concentration in Peru by size of Taxpayers
Source: STA Peru 2009

When a medium or small-sized seller does not pay the VAT, the STA may suspect that part or all of its transactions do not support real transactions, but there is not more prior information to prove it. Besides, most of the time auditing a small-sized firm has a negative cost/benefit. For the same reason, auditing the buyer would be desirable only when the added amount of suspicious transactions[2] makes it profitable. Therefore, small-sized evaders may continue operating for long time before being caught and/or punished.

Das-Gupta and Gang (1996) introduce the "Transactions Matching" technique to the conventional analysis. They point out that for VAT the matching of purchase and sale invoices is an important enforcement technique. Furthermore, they assure that transactions' matching has proved to have very different effects from auditing, and even when auditing alone is unable to induce non-zero taxpayer reports, a suitable intensive transaction matching may induce truthful reporting.

---

2   Including other transactions with other small-sized taxpayers that do not pay VAT.

Obviously, a tax enforcement mechanism may represent an additional barrier or cost of entry into a market. Indeed, Auriol and Warlters (2005) assure that informal economy in developing countries is largely a result of high fixed formal sector entry costs. However, despite the likely long-term negative effects of enforcement mechanisms on informality, we focus exclusively on the incentives that a mechanism like the DTR provides to VAT's compliance.

The Detailed Transactions Report (DTR) is a transaction-matching program established by Peru's Tax Administration Service (STA) that requires an important group of Peruvian taxpayers to deliver an annual database with sales and purchases detailed by clients and providers; this report is additional to the required annual tax statement[3]. Specifically, the DTR makes necessary the collusion between buyers and sellers, increasing the systemic risk as the number of taxpayers required also increase.

The DTR has to be delivered sometime after the annual VAT's statement, and those taxpayers who do not deliver it on time are fined, then those who do not deliver it after being fined are forbidden to continue operating.

Table 1 shows an example to understand how DTR works:

Table 1
DTR's information tabulated

Taxpayer	Sales		Purchases		Total Sales	Total Purchases
	To	USD	From	USD		
A	C	80	B	100	150	100
	D	70				
B	A	100	-	0	100	0
C	D	70	A	80	120	100
	E	50	D	20		
D	C	20	A	70	170	160
	E	60	C	70		
	F	90	G	20		
…….	……..		……..		……..	……..

There are two firms A and B, both required to report the DTR, and the STA totalizes all amounts reported to detect various modalities of VAT's

———

3   Which one does not have detailed but just totalized amounts?

evasion, but the most important data comes from comparing the total sums for each taxpayer with respect to its annual statement.

Table N° 2 shows that when we take a larger number of taxpayers, DTR becomes a powerful tool to detect VAT's fraud. Nevertheless, if a firm reports the DTR, but some of its clients and/or providers are not required to report it, it is not possible to match those specific transactions; e.g., if we do not demand DTR to taxpayers E, F and G, we will not be sure if: sales of taxpayers C and D, purchases of taxpayer D and sales and purchases of taxpayers E, F and G are accurate.

With this information, it is inferred that it would be difficult that taxpayers C and D report more sales than those they effectively made; we also may be tempted to think that if taxpayer G did not pay its VAT then taxpayer D could be overstating costs with credit not supported by real transactions. However, we do not have more prior information to confirm it.

Table 2
DTR's information totalized for more than two taxpayers

Taxpayer	Sales		Purchases	
	Informed by Others	Reported Annually	Informed by Others	Reported Annually
A	150	150	100	100
B	100	100	0	0
C	70	120	100	100
D	20	170	140	160

In Peru, the STA requires only 60 thousand taxpayers to report the DTR (a small percentage considering that approximately 560 thousand report the annual VAT's statement). Reporting taxpayers were and are still chosen based on their size which is highly correlated with their sales and purchases importance.

It should be clear that not paying VAT when due does not necessarily mean that a small- sized firm is issuing fake invoices. Indeed, it may be a simple lack of enforcement. However, VAT's evasion is around 34%[4] in Peru and this is the reason why it should be considered as a serious issue.

---

4   Measured by the IMF and the World Bank, it means that STA is not collecting 34% of the potential VAT.

Graetz, Reinganum and Wilde (1986) state that the main condition for the STA to audit is that the increase of revenues associated to uncovering a non-complier must exceed the audit cost. Since the STA may not have positive benefits on auditing when the amount of the probable non-real purchases is not enough to cover, at least, audit costs, there are incentives for the taxpayers to evade by splitting purchases to medium and small-sized providers who issue invoices that do not support real transactions.

If the list of taxpayers under VAT's scope is especially heterogeneous that most of them execute transactions for amounts that would not justify audits, and the STA cannot enforce regular VAT payments from all of them, at least three of the four elements mentioned by Franzoni (1999) to assure tax compliance, should not be considered.

The DTR aims to improve VAT's compliance in two different ways: first, by directly improving compliance due to the additional controls over those required to report[5] and second, by indirectly improving compliance as being informed in detail by other taxpayers' forces, then each taxpayer could report accurately to avoid an audit.

If the STA required DTR to all taxpayers under VAT's scope and performed a total collection process, the possibility of VAT's evasion would be considerably smaller. However, the DTR improves compliance only of those taxpayers whose providers and clients are all required, too. Therefore, firms have an incentive to overstate costs by splitting purchases to small-sized firms not required to deliver the DTR.

Even though a perfect tax collection may not be achieved, it is still possible to use the DTR as a tool to expulse those taxpayers whose only objective is issuing fake invoices from the system; e.g., if taxpayer G is issuing fake invoices, demanding DTR to G will obligate G to provide detailed information of all its "transactions". Then, it is very likely that if G is providing invoices not supported on real transactions, it will not report DTR, then the STA will be likely to fine and close it. Nevertheless, if G delivers DTR, increased risk and collusion costs play now major roles in its decision to evade.

Finally, since the intention of this article is only to prove that partially settled enforcement mechanisms have indeed negative effects on tax compliance, we do not analyze their economic effects on productivity or efficiency which can be certainly negative and may stimulate informality.

---

5   Not delivering the DTR implies fines and closures.

THE MODEL

We set a Game Theory Model in which there exist two active players: an Oligopolist and the STA; firm "m" is a passive player but will play actively in the extended version of this model. The Oligopolist (the purchaser) is a big-sized firm required to report the DTR, but its provider (the seller) "m" is a small-sized firm that is not required.

This is a game of incomplete information in which nature determines first the probability that any small-sized firm like "m" is issuing invoices that do not support real transactions or the percentage of firms like "m" doing that. In the meantime, collusion between the Oligopolist and "m" is not necessary since the STA cannot match their transactions.

The Oligopolist reports detailed purchases to "m" through the DTR, but the STA cannot match this information and suspects that the purchaser may be overstating costs since "m" did not paid VAT when due. However, the only way to prove evasion would be through auditing.

According to Alm (2012) there is no definitive explanation on why small or big-sized firms are willing to evade, and an attempting to provide one will clearly surpasses the scope and purpose of this article. Thus, we assume there are some small and big-sized firms willing to evade[6]. We will study the incentives that an enforcement mechanism like the DTR provides to them. This conveys us to assume that firms have pre-determined or exogenous probabilities of being evaders.

The basic assumptions of the model are:

The Oligopolist, due to the amount of its transactions, is always required to report the DTR while the competitive firm "m" that sells inputs to the Oligopolist, is not.

In general, these sellers (small sized firms) do not pay taxes when due, but just some of them issue invoices that do not support real transactions[7].

Taxpayers are risk-averse while STA is risk-neutral.

τ is the tax rate of a Value-Added-Tax $0 < \tau < 1$

---

6   This characteristic agrees exactly with empirical observations.

7   There is not a clear account of how many of these small-sized firms that do not pay VAT when due are created just for issuing invoices that do not support real transactions. However, as we said before, VAT's evasion is around 34% of its potential base according to 2009 estimates .

Since it is costly, STA may enforce VAT payments of the Oligopolist, but not of all small-sized firms like "m". A representative competitive firm "m" which sells inputs to the Oligopolist is such that:

$$x_m P(x_m) = M \qquad (1)$$

Where:

x_m is "m"'s output, $x_m > 0$

$P(x_m)$ is the price of x_m without tax

$x_m P(x_m)$ is "m"'s tax base. It will be called M

Firm "m" is a perfect competitor that may eventually sell all its production to only one purchaser (the Oligopolist).

On the other hand, an Oligopolist sells final products and its tax base is:

$$x\ P(x) \qquad (2)$$

All transactions are standardized to the unit.

α is the probability that any firm "m"'s sales do not support real transactions or the percentage of small-sized firms like "m" issuing these invoices, $0 \leq \alpha \leq 1$.

λ is the probability that a "dishonest" Oligopolist "buys" from "m" or the percentage of Oligopolists buying from "m" when "m" is issuing invoices that do not support real transactions, $0 \leq \lambda \leq 1$.

δ is the probability that an "honest" Oligopolist "buys" from "m" or the percentage of Oligopolists buying from "m" when "m" issues invoices that support real transactions. Henceforth, and without changes in the development and conclusions of this article, we assume that $\delta = 1$ or that an honest Oligopolist always purchases from "m" when "m" is issuing invoices that do support real transactions. Alternatively, when some of all firms type "m" issue invoices supporting real transactions, all of the "honest" Oligopolists in the market purchase from them.

θ is the percentage of M that a firm "m" issuing invoices that do support real transactions would collect for selling them, it is produced by the agreement of buyers and sellers[8], and it implies:

$$\theta M > \tau M$$

---

8 Empirical evidence from most audits suggests that θ is around 1 or 2 percentage points above τ, so that we consider it a parameter

Then:

$$\theta > \tau \tag{3}$$

Equation (3) means that what "m" expects to be paid for these invoices must be greater than the amount it would have to pay if audited. In other words this is "m"'s minimum incentive to cheat. In order to simplify, henceforth invoices that do not support real transactions will be called "fake invoices"[9].

β is the STA's fixed probability of auditing/detecting[10] VAT's evasion $0 \leq \beta \leq 1$

The STA has limited resources and it may audit only some big-sized firms; moreover, since the amount of transactions are in general small, and the Oligopolists are used to have minor percentages of their purchases from small-sized firms[11], we assume that auditing any Oligopolist due to these suspicious transactions reports approximately the same expected benefit to the STA. Therefore, the probability of being audited by this reason is exogenous and fixed.

F is the fine that the STA imposes to the Oligopolist when it detects invoices are fake.

$$F[M] \tag{4}$$

Where F > 0

Notice that the STA first observes the purchases to firm "m" reported in detail by the Oligopolist, and notice that "m" did not pay VAT; however, it is not sure if these transactions are fake, then a conjecture (belief) should be designed about the nature of "m"'s sales to decide if the Oligopolist is audited or not.

There is a fixed cost of auditing, named "*a*"

First, nature determines if firm "m" issues fake invoices or the percentage of firms like "m" is doing the same. Then, the Oligopolist decides whether to buy from "m" or not (detailed-reporting-collusion is not necessary since "m" does not report DTR).

---

9   Indeed, these invoices are real and authorized by the STA, there are also "fake" or not authorized invoices in the market, but they constitute a small percentage of STA's finds.

10   We assume that auditing allows the STA to detect evasion in all cases.

11   However, the totalized amounts of these suspicious purchases are very important.

Finally, the STA observes detailed purchases of the Oligopolist to "m", then it makes conjectures about the nature of these transactions and decides to audit or not.
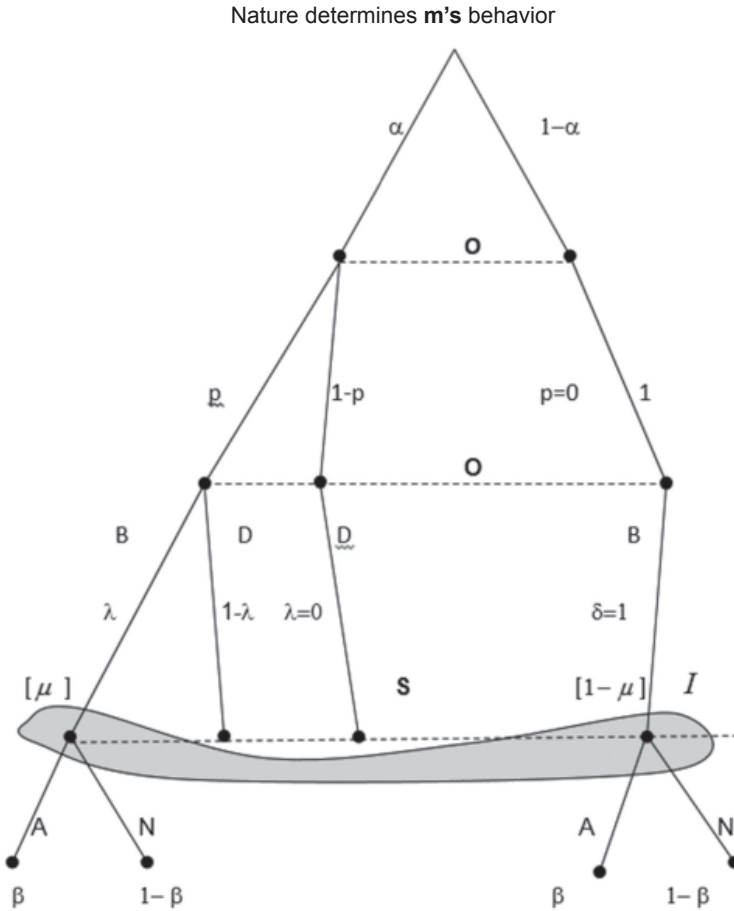
The Game Tree is:

Nature determines **m's** behavior

Figure 2: Structure of the game

Where:

α : is the probability that "m"'s sales are fake or the percentage of firms like "m" sells fake invoices

O: is the Oligopolist

p: Is the percentage of Oligopolists who may decide to evade or the probability that an Oligopolist is "dishonest".

1-p: is the percentage of Oligopolists who decide not to evade under any circumstance or the probability that an Oligopolist is "honest".

S: is the STA that does not know (before auditing) if purchases reported are based on real transactions or not.

λ : is the Probability that a dishonest Oligopolist purchases to the provider or the percentage of these Oligopolists that purchases from firm "m" when "m" is issuing fake invoices.

δ : is the Probability that an honest Oligopolist purchases to the provider or the percentage of Oligopolists that purchases from firm "m" when "m" is issuing real invoices.

The tree shows that the probability of purchasing fake invoices given that the Oligopolist is honest, is zero $(\lambda = 0)$.

For the same reason, and in order to simplify the analysis without affecting our final results, we assume that a dishonest Oligopolist has zero purchase probability (p=0) from "m" given that "m" issues real invoices.

Finally, in the case of the honest Oligopolist, a purchase probability from "m" given that invoices are real, is one $(\delta = 1)$.

β : is the fixed probability of audit explained before.

Notice that this is a dynamic game with incomplete information (payoffs unknown) but following Harsanyi (1967) it is possible to solve it as a game with imperfect information (history unknown)

In this case nature reveals the type of "m" to the Oligopolist, but does not reveal it to the STA, then we may model this game as of imperfect information since at the time the STA has to play, it does not know if purchases reported by the Oligopolist are real or not.

Shaded area on the tree graphic shows the STA's information set I.

The STA needs to determine whether to audit or not the Oligopolist and then it needs first to make an assumption about the Distribution of Oligopolists according to their probability of buying from "m" given "m" is issuing fake invoices.

Assumptions using Bayes' rule are:

$$\mu = \frac{\alpha p \lambda}{1 - \alpha(1 - p\lambda)}$$

μ: is the probability that an Oligopolist "purchases" from a provider given that this provider is issuing "fake invoices". It reflects STA's belief about the Oligopolist's nature (if it is willing or not to buy fake invoices)

$$1 - \mu = \frac{1 - \alpha}{1 - \alpha(1 - p\lambda)}$$

1-μ: is STA's conjecture of the probability that an Oligopolist who is not willing to purchase fake invoices, purchases from a provider given that this provider is issuing real invoices.

A: is the STA's action of Auditing

N: is the STA's action of not auditing

B: is the Oligopolist's action of Purchasing from "m"

D: is the Oligopolist's action of Not Purchasing from "m"

Table N° 3 shows Marginal benefits of the players.

Rows represent Oligopolist's strategies and columns the STA's strategies.

$(B_p; \alpha)$ is the Dishonest Oligopolist's decision of purchasing from "m" given that "m" is issuing fake invoices.

$(B_{1-p}; 1-\alpha)$ is the Honest Oligopolist's decision of purchasing from "m" given that "m" is issuing real invoices.

$(D_p; \alpha)$ is the Dishonest Oligopolist's decision of Not purchasing from "m" given that "m" is issuing fake invoices.

$(D_{1-p}; \alpha)$ is the Honest Oligopolist's decision of Not purchasing from "m" given that "m" is issuing fake invoices.

FINDING PERFECT BAYESIAN EQUILIBRIA

STA's expected payoff from choosing A (audit) is:

$$\mu\left[\tau x P(x) + F(M) - a\right] + (1-\mu)\left[\tau x P(x) - \tau M - a\right]$$

From choosing N (Not audit) is:

$$\tau\left[x P(x) - M\right]$$

To audit, the STA needs that A > N which implies:

$$\mu > \frac{a}{(F+\tau)M} \tag{5}$$

Table 3
Strategic form representation and payoffs (Marginal benefits of auditing for the STA and purchasing for the Oligopolist

	A		N	
	Oligopolist	STA	Oligopolist	STA
$(\mathbf{B_p};\ \alpha\ )$	$(1-\tau)xP(x)-(F+\theta)M$	$\tau x P(x)+F(M)-a$	$(1-\tau)xP(x)+(1-\tau-\theta)M$	$\tau x P(x)-\tau M$
$(\mathbf{B_{1-p}};1-\alpha\ )$	$(1-\tau)xP(x)-(1-\tau)M$	$\tau x P(x)-\tau M-a$	$(1-\tau)xP(x)-(1-\tau)M$	$\tau x P(x)-\tau M$
$(\mathbf{D_p};\ \alpha\ )$	$(1-\tau)xP(x)$	$\tau x P(x)-a$	$(1-\tau)xP(x)$	$\tau x P(x)$
$(\mathbf{D_{1-p}};\alpha\ )$	$(1-\tau)xP(x)$	$\tau x P(x)-a$	$(1-\tau)xP(x)$	$\tau x P(x)$

If the dishonest Oligopolist has "bought" fake invoices from "m", and STA has chosen to audit, the conjecture μ=1 turns to be true and it implies that:

$$M > \frac{a}{(F + \tau)}$$

This expression shows the minimum amount that justifies an audit for the STA.

However, we stated that amounts like M are too small that do not justify an audit so that we will assume for the time being that:

$$M > \frac{a}{(F + \tau)} \qquad (6)$$

If equation (6) holds then the STA's best choice is "N" which implies A < N or:

$$\mu < \frac{a}{(F + \tau)M}$$

If the STA chooses not to audit then the Dishonest Oligopolist has to choose between B and D.

In case of choosing B, the expected benefit is:

$$\alpha p \lambda \big[(1 - \tau)xP(x) + (1 - \tau - \theta)M\big]$$

If choosing D is:

$$\alpha p \lambda \big[(1 - \tau)xP(x)\big]$$

If we assume that B>D:

$$\alpha p \lambda \big[(1 - \tau)xP(x) + (1 - \tau - \theta)M\big] > \alpha p \lambda \big[(1 - \tau)xP(x)\big]$$

$$(1 - \tau - \theta)M > 0$$

Implying that:

$$1 - \tau > 0 \qquad (7)$$

Or that:

$$M > 0 \qquad (8)$$

Equation (7) implies that for each transaction, after paying the tax, the margin must be greater than the payment made to firm "m" for issuing fake invoices. Since this equation supports Oligopolist's cheating

decision[12] we will assume this is true henceforth. For the same reason, equation (8) holds to support this profile.

Thus, in this particular case, given that the STA chose not to audit, a Dishonest Oligopolist will choose B.

With profile (B,N) Bayes' rule turns one μ=1, then:

$$1 < \frac{a}{(F + \tau)M}$$

Or that:

$$a > (F + \tau)M \tag{9}$$

Equation (9) is equal to our assumption stated in (6). Therefore, the Profile (B,N) is a PBE if equation (8) holds.

This unique PBE means that the Dishonest Oligopolist chooses purchasing from "m" given that "m" is issuing fake invoices, and the STA chooses not to audit.

This equilibrium predicts that a Dishonest Oligopolist will prefer to split purchases to medium or small-sized providers like "m" that issue fake invoices and are not required to deliver DTR, and the STA will choose not to audit since the audit cost is greater than the expected increase in revenues of detecting fake transactions (fine plus tax)

The only policy rule the STA may use in this equilibrium is "F"[13]; the other way of making profitable an audit would be by reducing its cost which may create a trade-off between quality and efficiency.

EXTENSION TO THE ORIGINAL MODEL

It would be interesting to explore an extension to the original model in which DTR is required to all firms under VAT's scope to determine if there are positive effects on compliance after doing it.

All firms (including those like "m") are required now to deliver the DTR. Then, there is a collusion cost added to the game:

$$(\theta M - cM > \tau M \tag{10}$$

---

12 A dishonest Oligopolist would not pay more for a fake invoice than the eventual margin after tax on it.

13 In Peru as in most countries of Latin America moving the tax rate is not a policy controlled by the STA

Equation (10) shows the net benefit of issuing fake invoices for a determined provider "m"; the left-hand side shows its benefit (payment) and the right-hand side shows its cost.

We have c > 0 as the cost of collusion for "m" when required to deliver DTR and assume that the cost of collusion for the Oligopolist is incorporated in θ. Then, following (10), the original condition (3) turns to:

$$\theta - c > \tau \tag{11}$$

Equation (11) shows the new minimum incentive to cheat.

There are now three active players in the game, and the STA may still audit the Oligopolist, but not "m" since it is expensive. However, after auditing the Oligopolist it may fine both taxpayers in case of fake invoices.

The Game Tree is still the same but now the first move is made by "m" since it has now the alternative of choosing between issuing fake or unreal invoices (U) with probability "α" or issuing real invoices (R) with probability "$1 - \alpha$".

Even though "m" is required to deliver the DTR it does not pay VAT when due and this is the reason why the STA suspects transactions may be fake.

Since "m" does not pay VAT when due, if the STA audits the Oligopolist and transactions are real, the STA collects VAT from "m" in order to validate the Oligopolist's credit.

Benefits are shown in Table N°4.

As we may see, there are just small changes with respect to the original setting. The most important changes are the inclusion of "m"'s marginal benefits and the inclusion of a double fine (to the Oligopolist and to "m") in the STA's expected benefit if transactions are fake. The latter is important because the DTR is established by law and in this sense the STA may better control "m".

This leads us to change the assumption stated in equation (6), now with a double fine this equation turns to:

$$M > \frac{a}{(2F + \tau)} \tag{12}$$

Now we assume equation (12) holds.

Table N° 4 shows the marginal benefits in this extended version of the game.

Table 4
Strategic form representation and payoffs (Marginal benefits of the STA, the Oligopolist and firm "m")

	A			N		
	Oligopolist	**STA**	**m**	**Oligopolist**	**STA**	**m**
$(\mathbf{B_p};\alpha)$	$(1-\tau)xP(x)-(F+\theta)M$	$\tau x P(x)+2F(M)-a$	$(\theta-c-F)M$	$(1-\tau)xP(x)+(1-\tau-\theta)M$	$\tau x P(x)-\tau M$	$(\theta-c)M$
$(\mathbf{B_{1\text{-}p}};1-\alpha)$	$(1-\tau)xP(x)-(1-\tau)M$	$\tau x P(x)-\tau M-a$	$(1-\tau)M$	$(1-\tau)xP(x)-(1-\tau)M$	$\tau x P(x)-\tau M$	M
$(\mathbf{D_p};\alpha)$	$(1-\tau)xP(x)$	$\tau x P(x)-a$	0	$(1-\tau)xP(x)$	$\tau x P(x)$	0
$(\mathbf{D_{1\text{-}p}};\alpha)$	$(1-\tau)xP(x)$	$\tau x P(x)-a$	0	$(1-\tau)xP(x)$	$\tau x P(x)$	0

FINDING PERFECT BAYESIAN EQUILIBRIA

Let us find the Perfect Bayesian Equilibria (PBE) of this game.

STA's expected payoff from choosing A (audit) is:

$$\mu\left[\tau x P(x)+2F(M)-a\right]+(1-\mu)\left[\tau x P(x)-\tau M-a\right]$$

Notice that STA may now impose a double fine, one to the Oligopolist and one to "m" in case of fake invoices.

If choosing N (Not audit):

$$\tau\left[x P(x)-M\right]$$

Therefore, when choosing "A" over "N" we obtain STA's Best Response function:

$$\beta(A)\begin{cases}1, if, \mu > \dfrac{a}{(2F+\tau)M}\\[3mm]0, if, \mu < \dfrac{a}{(2F+\tau)M}\\[3mm]\left[0,1\right], Otherwise\end{cases}\tag{13}$$

Let us suppose that:

$$\mu > \frac{a}{(2F+\tau)M}\tag{14}$$

Then, the STA is sure to choose A (audit) according to its information and the Dishonest Oligopolist (p) would have to choose between Purchasing (B) and Not Purchasing (D) from "m".

Expected Benefit of Purchasing (B):

$$\alpha p\left[(1-\tau)x P(x)-(F+\theta)M\right]$$

Expected Benefit of Not Purchasing (D):

$$\alpha p\left[(1-\tau)x P(x)\right]$$

It is clear that B < D then the dishonest Oligopolist will choose D.

With choices D and A, player "m"s expected benefits of choosing are:

U = 0

R = 0

Let us assume "m" chooses "U".

The profile (U,D,A) makes $\mu = 0$

Also making:

$0 > a$

Which contradicts the assumptions, then the evaluation $\{(U,D,A);$ $\mu = 0,\ 1 - \mu = 1\}$ is not a PBE.

Let us assume "m" chooses "R".

The profile (R,D,A) makes $\mu = 0$

Also making:

$0 > a$

Which also contradicts the assumptions, then the evaluation $\{(R,D,A);$ $\mu = 0,\ 1 - \mu = 1\}$ is not a PBE.

Let us suppose now that:

$$\mu < \frac{a}{(2F + \tau)M} \tag{15}$$

Then the STA is sure to choose N (Not audit) and the Dishonest Oligopolist has to choose between B and D.

The expected benefit in case of choosing B is:

$$\alpha p\big[(1 - \tau)xP(x) + (1 - \tau - \theta)M\big]$$

If choosing D:

$$\alpha p\big[(1 - \tau)xP(x)\big]$$

Since we assume (7) is true, then it is clear that B > D; therefore, the Oligopolist will choose B.

With decisions (B,N), $\lambda = 1$ and $\delta = 1$, "m"'s expected benefit in case of choosing U is:

$$\alpha p(\theta - c)M$$

If choosing R:

$$(1 - \alpha)M$$

Assuming U > R we obtain m's best response function:

$$(\theta - c)_{\alpha p} \begin{cases} 1,\ if,\ \theta - c > \dfrac{1 - \alpha}{\alpha p} \\[2mm] 0,\ if,\ \theta - c < \dfrac{1 - \alpha}{\alpha p} \\[2mm] [0,\ 1],\ Otherwise \end{cases} \tag{16}$$

Let us assume that "m" chooses U or that:

$$\theta - c > \frac{1-\alpha}{\alpha p} \tag{17}$$

It means that "m" will issue unreal or fake invoices when its expected benefit of doing it is higher than the ratio of "Honest" versus "Dishonest" Taxpayers.

Then the profile (U,B,N) makes $\mu = 1$ or:

$$M < \frac{a}{(2F+\tau)} \tag{18}$$

But (18) contradicts the assumption stated with equation (12)

Therefore, the evaluation $\{(U,B,N);\ \mu = 1,\ 1-\mu = 0\}$ is not a PBE.

Let us assume now that "m" chooses R or that:

$$\theta - c < \frac{1-\alpha}{\alpha p} \tag{19}$$

With the profile (R,B,N) Bayes' rule makes $\mu = 1,$ thus:

$$M < \frac{a}{(2F+\tau)} \tag{20}$$

Equation (20) also contradicts the assumption stated with equation (12). Therefore, the evaluation $\{(R,B,N);\ \mu = 1,\ 1-\mu = 0\}$ is not a PBE.

Now, let us assume that "m" is mixing or that:

$$\theta - c = \frac{1-\alpha}{\alpha p} \tag{21}$$

With the profile (mixing,B,N) Bayes' Rule turns $0 < \mu < 1$ then:

$$\mu M < \frac{a}{(2F+\tau)} \tag{22}$$

Therefore, the evaluation $\{(mixing,B,N);\ 0 < \mu < 1\}$ is a PBE if conditions (21) and (22) hold. For μ small enough, (22) allows (12) to hold.

Let us suppose now that STA is mixing strategies:

$$\mu < \frac{a}{(2F+\tau)M} \tag{23}$$

The STA will choose A with probability expressed by (23)

Dishonest Oligopolist's expected benefit in case of choosing B is:

$$\alpha p\left\{\beta[(1-\tau)xP(x)-(F+\theta)M]+(1-\beta)\left[(1-\tau)xP(x)+(1-\tau-\theta)M\right]\right\}$$

If choosing D:

$$\alpha p\left[(1-\tau)xP(x)\right]$$

Let us assume that B > D then the dishonest Oligopolist's best response function is:

$$\beta \begin{cases} 1, if, \beta < \dfrac{1-\tau-\theta}{1-\tau+F} \\ 0, if, \beta > \dfrac{1-\tau-\theta}{1-\tau+F} \\ \left[0,1\right], Otherwise \end{cases} \tag{24}$$

Let us suppose that:

$$\beta < \dfrac{1-\tau-\theta}{1-\tau+F} \tag{25}$$

Equation (25) means that the Dishonest Oligopolist will choose B since the probability of auditing is less than the ratio of Oligopolist's expenditures with respect to STA's revenues in case of fake transactions.

If the Dishonest Oligopolist chooses definitely B then "m'"'s benefit in case of choosing U is:

$$\alpha p \lambda \left\{\beta[(\theta-c-F)M]+(1-\beta)[(\theta-c)M]\right\}$$

If choosing R:

$$(1-\alpha)(1-p)\delta\left\{\beta(1-\tau)M+(1-\beta)M]\right\}$$

Let us assume U > R then m's Best Response function is:

$$\dfrac{(\theta-c)-\beta F}{1-\beta\tau} \begin{cases} 1, if, \dfrac{(\theta-c)-\beta F}{1-\beta\tau} > \dfrac{1-\alpha}{\alpha p} \\ 0, if, \dfrac{(\theta-c)-\beta F}{1-\beta\tau} < \dfrac{1-\alpha}{\alpha p} \\ \left[0,1\right], Otherwise \end{cases} \tag{26}$$

Let us assume "m" chooses U or that:

$$\dfrac{(\theta-c)-\beta F}{1-\beta\tau} > \dfrac{1-\alpha}{\alpha p} \tag{27}$$

Equation (27) says that "m" chooses "U" if its net benefit ratio coming from issuing fake invoices is higher than the ratio of honest-dishonest taxpayers.

The profile (U,B,mixing) makes $\mu = 1$ or that:

$$M = \frac{a}{(2F + \tau)} \qquad (28)$$

Which contradicts the assumption stated in equation (12)

Therefore, the evaluation $\{(U,B,mixing); \ \mu = 1, 1 - \mu = 0\}$ is not a PBE.

Let us suppose now that "m" chooses R or that:

$$\frac{(\theta - c) - \beta F}{1 - \beta\tau} < \frac{1 - \alpha}{\alpha p} \qquad (29)$$

The profile (R,B,mixing) makes $\mu = 1$ or that:

$$M = \frac{a}{(2F + \tau)} \qquad (30)$$

Which also contradicts the assumption stated in equation (12)

Therefore, the evaluation $\{(R,B,mixing); \ \mu = 1, 1 - \mu = 0\}$ is not a PBE.

Let us assume "m" mixes or that:

$$\frac{(\theta - c) - \beta F}{1 - \beta\tau} = \frac{1 - \alpha}{\alpha p} \qquad (31)$$

The profile (mixing, B, mixing) makes $\mu > 0$ or that:

$$\mu M < \frac{a}{(2F + \tau)} \qquad (32)$$

That for μ small enough allows equation (12) to hold.

Therefore, the evaluation $\{(mixing,B,mixing); \ 0 < \mu < 1\}$ is a PBE requiring conditions (25), (31) and (32) to hold.

Let us suppose that:

$$\beta > \frac{1 - \tau - \theta}{1 - \tau + F} \qquad (33)$$

The dishonest Oligopolist will definitely choose D then "m"'s benefit is:

$U = 0$

$R = 0$

The dishonest Oligopolist choosing D, and the STA mixing the Profiles (U,D,mixing), (R,D,mixing) and (mixing,D,mixing) make $\mu = 0$, it implies:

$a = 0$

Therefore, their related evaluations are not PBE.

Let us suppose that:

$$\beta = \frac{1 - \tau - \theta}{1 - \tau + F}$$

(34)

The Dishonest Oligopolist is mixing. Then m's benefit in case of choosing U is:

$$\alpha p \lambda \{\beta[(\theta - c - F)M] + (1 - \beta)[(\theta - c)M]\} + \alpha p (1 - \lambda)\{0\}$$

If choosing R:

$$(1 - \alpha)(1 - p)\delta \{\beta(1 - \tau)M + (1 - \beta)M\}$$

Let us assume U > R, then m's Best Response function is:

$$\frac{(\theta - c) - \beta F}{1 - \beta \tau} \begin{cases} 1, if, \dfrac{(\theta - c) - \beta F}{1 - \beta \tau} > \dfrac{1 - \alpha}{\alpha p \lambda} \\[2mm] 0, if, \dfrac{(\theta - c) - \beta F}{1 - \beta \tau} < \dfrac{1 - \alpha}{\alpha p \lambda} \\[2mm] \left[0, 1\right], Otherwise \end{cases}$$

(35)

Let us assume "m" chooses U or that:

$$\frac{(\theta - c) - \beta F}{1 - \beta \tau} > \frac{1 - \alpha}{\alpha p \lambda}$$

(36)

Equation (36) shows that "m" chooses "U" if its net benefit ratio of issuing fake invoices is higher than the ratio of honest-dishonest taxpayers. Since the Oligopolist is now mixing the probability "λ", it is not one like in equation (26).

The profile (U,mixing,mixing) makes $\mu = 1$ or that:

$$M = \frac{a}{(2F + \tau)}$$

(37)

Which contradicts the assumption stated in equation (12)

Therefore, the evaluation $\{(U,mixing,mixing); \quad \mu = 1, 1 - \mu = 0\}$ is not a PBE.

Let us assume "m" chooses R or that:

$$\frac{(\theta - c) - \beta F}{1 - \beta \tau} < \frac{1 - \alpha}{\alpha p \lambda} \qquad (38)$$

The profile (R,mixing,mixing) makes $\mu = 0$ or that:

$$a = 0 \qquad (39)$$

Which contradicts the initial assumptions.

Therefore, the evaluation $\{(\text{R,mixing,mixing}); \quad \mu = 1, 1 - \mu = 0\}$ is not a PBE.

Let us assume "m" mixes or that:

$$\frac{(\theta - c) - \beta F}{1 - \beta \tau} = \frac{1 - \alpha}{\alpha p \lambda} \qquad (40)$$

The profile (mixing,mixing,mixing) makes $0 < \mu < 1$ or that:

$$\mu M = \frac{a}{(2F + \tau)} \qquad (41)$$

For μ small enough it allows equation (12) to hold.

Therefore, the evaluation $\{(\text{mixing,mixing,mixing}); \ 0 < \mu < 1\}$ is a PBE requiring conditions (34), (40) and (41) to hold.

Consequently, we have 3 possible PBE in this game. Let us analyze them:

The first PBE is the evaluation $\{(\text{mixing,B,N}); \ 0 < \mu < 1\}$ which means that firm "m" randomizes on issuing fake and real invoices, the Dishonest Oligopolist chooses to purchase from "m" and the STA chooses not to audit, then this PBE requires that:

The expected benefit of issuing fake invoices for "m" is equal to the ratio of honest versus dishonest taxpayers:

$$\theta - c = \frac{1 - \alpha}{\alpha p}$$

The expected amount of fake credit to be audited is smaller than the cost / benefit ratio of the audit:

$$\mu M < \frac{a}{(2F + \tau)}$$

This equilibrium predicts that if the expected benefit of issuing fake invoices for "m" is equal to the ratio of honest versus dishonest taxpayers, it becomes an incentive to evade for "m". In other words, if there are enough evaders in the market making the ratio lower, a particular firm

like "m" would be more inclined to evade or there would be more firms like "m" inclined to evade.

This result is not as bad as the one obtained in the original setting; not only since the collusion cost may play a new and major role in m's choice, but also by the possibility of improving compliance by using other alternative measures.

If the STA can directly or indirectly increase "c" in order to reduce m's net benefit of evading, this negative result may be eventually avoided. However, increasing the collusion cost may have worse effects on compliance in the long term as we pointed out before.

Nevertheless, a kind of "behavioral" factor plays a major role here. Indeed, "m" is inclined to evade when it looks that most taxpayers evade. Alternative policy rules like school education for kids on the benefits of taxation, improving the communication with taxpayers or even better just simplifying the system would obtain much better results in the long-run aggregate behavior of taxpayers as stated by Alm (2012) and displayed before in the literature review.

Finally, if the expected amount of fake credit to be audited is smaller than the cost / benefit ratio of the audit, the STA would choose not to audit. Nevertheless, since we assume that (12) holds, it means that the double fine would induce the STA to consider an audit profitable enough to assign at least a small probability to its choice of audit; this idea conveys us to a necessary equilibrium refinement.

The second PBE is the evaluation {(mixing,B,mixing); $0 < \mu < 1$} which means that the firm "m" randomizes on issuing fake and real invoices, the Dishonest Oligopolist chooses to purchase from "m" and the STA randomizes on auditing or not; this PBE requires that:

The net rate of return of issuing fake invoices for "m" is equal to the ratio of dishonest / honest taxpayers:

$$\frac{(\theta - c) - \beta F}{1 - \beta \tau} = \frac{1 - \alpha}{\alpha p}$$

The audit probability is smaller than the ratio of net margin after buying fake invoices / margin plus fine:

$$\beta < \frac{1 - \tau - \theta}{1 - \tau + F}$$

The expected amount of fake credit to be audited is equal to the cost / benefit ratio of the audit:

$$\mu M = \frac{a}{(2F + \tau)}$$

This equilibrium predicts that "m" will randomize on issuing or not fake invoices or that there exist firms like "m" issuing fake invoices and others issuing real invoices at the same time. The dishonest Oligopolist will definitely buy from "m" when "m" issues fake invoices or all Oligopolists willing to evade will purchase from firms like "m" when they issue fake invoices. Finally, the STA will randomize on auditing or not.

There is space for STA's policies in this environment too; e.g., if the STA is able to fix a high enough fine "F" even a small fixed probability of audit may induce the dishonest Oligopolist to avoid purchasing fake invoices. However, as Alm (2012) has pointed out, most empirical studies have found that compliance increases, but only slightly with increases in the fine rate on unpaid taxes.

On the other hand, the same behavioral factor mentioned in the first PBE is also important in m's choice.

As in the first equilibrium, the dishonest Oligopolist's definite choice of "B" appears not to be optimal when the STA is assigning a positive probability to audit, then this equilibrium may also be analyzed with a stricter concept or a PBE's refinement.

The third PBE is the evaluation {(mixing,mixing,mixing); $0 < \mu < 1$} which means that firm "m" randomizes on issuing fake and real invoices, the Dishonest Oligopolist randomizes on buying and not buying fake invoices from "m", and the STA randomizes on auditing or not. Alternatively, it means that some percentage of firms like "m" chooses to evade and some percentage of the Oligopolists, even willing to evade, chooses not to do it. This PBE requires that:

The net rate of return of issuing fake invoices for "m" is equal to the ratio of dishonest / honest taxpayers:

$$\frac{(\theta - c) - \beta F}{1 - \beta \tau} = \frac{1 - \alpha}{\alpha p \lambda}$$

The audit probability is equal to the ratio of net margin after purchasing fake invoices / margin plus fine:

$$\beta = \frac{1 - \tau - \theta}{1 - \tau + F}$$

The expected amount of fake credit to be audited is equal to the cost / benefit ratio of the audit:

$$\mu M = \frac{a}{(2F + \tau)}$$

As in the other equilibria there is also space for STA's policies in this environment; the dishonest Oligopolist's decision of acting strategically to the randomization of "m" and the STA is what makes it more complete.

This Oligopolist's choice is a better response to small deviations from optimal decisions of the other players. Indeed, it definitely fulfills stricter requirements of equilibria than that of choosing "B".

TREMBLING HAND PERFECTION

In a PBE each player's equilibrium strategy is the best response to the other Player's strategies. However, a Trembling Hand Perfect Equilibrium (THPE) additionally imposes that the PBE should be robust to small perturbations (or mistakes) in the strategies played by other players. Hence, a THPE equilibrium is robust to an arbitrarily small and strictly positive probability that other players play other pure strategies, that each player's equilibrium strategy is still the best response to other players' perturbed strategies.

By definition any completely mixed strategy which is a PBE is always THPE.

Let us refine our former three PBE to find out if any or some of them are THPE.

The first PBE is the evaluation {(mixing,B,N); $0 < \mu < 1$}

Dishonest Oligopolist's optimal choice when the STA chooses "N" and "m" is mixing is "B". However, since (12) is an assumption, the STA would be better off by assigning at least a minimal, but strictly positive probability to its audit (A) choice when the Oligopolist is choosing "B" and "m" is mixing. Thus, STA's definite choice of "N" is not optimal and this evaluation is not a THPE.

The second PBE is the evaluation {(mixing,B,mixing); $0 < \mu < 1$}

Dishonest Oligopolist's optimal choice when "m" is mixing would be "B". However, since STA is also mixing, the Oligopolist would be better off by assigning at least a minimal, but strictly positive probability to its Not purchasing (D) choice. Thus, the dishonest Oligopolist's definite choice of "B" is not optimal and this evaluation is not a THPE, either.

The third PBE is the evaluation {(mixing,mixing,mixing); $0 < \mu < 1$}

Since by definition a completely mixed strategy which is a PBE is always THPE, this is the only THPE of the extended game.

In this equilibrium, the dishonest Oligopolist is better off by assigning at least a minimal, but strictly positive probability to its Not purchasing (D) choice, he is acting strategically to the strategies used by "m" and the STA, and each of these two are also acting strategically to the other players' strategies.

Here "m" is looking at its net rate of return in comparison with the behavior of all taxpayers in the market; the dishonest Oligopolist is evaluating the probability of being audited, and the STA evaluates the expected profitability of auditing.

CONCLUSIONS

Adequate VAT's collection and additional enforcement mechanisms like DTR would assure VAT's compliance when working together. However, getting a complete coverage in VAT's collection is difficult in a highly heterogeneous market like the Peruvian market. Therefore, we have reasons to believe that it would be beneficial to require the DTR to all taxpayers under VAT's scope.

If VAT's evasion from firms like "m" is as simple as it appears to be, many of them will choose not to report the DTR and the STA may legally fine and even close them definitely. On the contrary, if they choose to deliver the DTR they will have to evaluate again their net benefits. When firm "m" is required to deliver the DTR (forced to play) the effectiveness of the STA's policies increases considerably even though recent literature shows that fines and punishments are not as effective as the Allingham and Sandmo's portfolio approach suggests.

It is clear that in the reduced version of the model the only Equilibrium Profile (B,N) provides little room for STA's policies. On the other hand, equilibrium in the extended version implies that firm "m" and the Oligopolist evaluate their decisions regarding collusion costs, fines, the audit probability and the ratio of honest versus dishonest taxpayers. There is now enough space for using common policies and, even better, other institutional measures like education, mass communication or tax reforms oriented to simplify the system as powerful instruments to disincentive evaders.

Concretely, the proposal of requiring the DTR to all taxpayers under VAT's scope appears to be faster and cheaper than implementing a complete and more accurate tax collection to the large number of medium and small-sized firms in the Peruvian market. The most important lesson of this article is that partially settled enforcement mechanisms may be not only useless, but also harmful to improve compliance.

It is necessary to remember, as we pointed out before, that no references to economic efficiency or distortions caused by enforcement mechanisms are implied in this article. Indeed, barriers to access to the market, as those pointed out by Auriol and Warlters (2005) may stimulate underground economy and have been extensively studied in recent literature. Certainly, both theoretical and empirical effects of VAT's enforcement mechanisms on efficiency could be studied later as a manner of improving this partial effort.

REFERENCES

Alm, J. (2012). Measuring, explaining, and controlling tax evasion: lessons from theory, experiments, and field studies. *International Tax Public Finance*, *19*, 54-77.

Allingham, M. & Sandmo, A. (1972). Income Tax Evasion: A Theoretical Analysis. *Journal of Public Economics*, (1), 323-338.

Auriol, E. & Walters, M. (2005) Taxation Base in Developing Countries. *Journal of Public Economics 89*, 625-646.

Franzoni, L. A. (1999). Tax Evasion and Tax Compliance (September 1, 1998). Encyclopedia of Law and Economics, Vol. 4, pp. 51-94. Bouckaert, B. and De Geest, G. (Eds). UK: and Belgium: Edward Elgar.

Galiani, S. & Weinschelbaumf, F. (2011). Modelling Informality Formally: Households and Firms. *Economic Inquiry, 50*(3), 821-838.

Gang, I. N. & Das-Gupta, A. (1996) Value Added Tax Evasion, Auditing and Transactions Matching. Rutgers University (WP 96-07). Available at SSRN: http://ssrn.com/abstract=15026

Graetz, M. J., Reinganum, J. F. & Wilde, L. L. (1986). The Tax Compliance Game: Toward an Interactive Theory of Law Enforcement. *Journal of Law, Economics, & Organization, 2*(1), 1-32.

Harsanyi, J. C. (1967). Games with Incomplete Information Played by "Bayesian" Players, I-III. Part I. The Basic Model. *Management Science, 14*(3), 159-182.

Mittone, L. (2002). Motivations and collusion among agents in the evasion of indirect taxes: an experimental approach. Department of Economics and Computable and Experimental Economics Laboratory (CEEL), University of Trento, Working Paper. Available at: http://eprints.biblio.unitn.it/50/1/CEEL02_04.pdf