

April 2020

# Working Paper

**004.2020**

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## **A Tragic Solution to the Collective Action Problem: Implications for Corruption, Conflict and Inequality**

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# Firms and Cities Transition Towards Sustainability

## Series Editor: Stefano Pareglio

### A Tragic Solution to the Collective Action Problem: Implications for Corruption, Conflict and Inequality

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#### Summary

We study the role of an enforcer in the effectiveness of selective incentives in solving the collective action problem when groups take part in a contest. Cost functions exhibit constant elasticity of marginal effort costs. If prize valuations are homogeneous, our source of heterogeneity induces full cost-sharing and the first-best individual contributions; further, the group probability of winning goes up. With heterogeneity in prize valuations, an increase in the effectiveness of the enforcer in conflict increases the group probability of winning only if the prize valuation of the enforcer is lower than the Lehmer mean of those of the other players; however, the induced partial cost sharing is not group efficient. If effectiveness "tends to infinity", the collective action problem is solved with partial cost-sharing if that prize valuation is not too low. Tragically, if productivity is low (if the prize is private in our set up) this occurs with corrupt coalitions which have been shown to form together with conflict and inequality endogenously; otherwise, this occurs with non corrupt coalitions. Further, even if such valuation is too low the group winning probability goes up. In this latter case, over cost-sharing yields group efficiency.

**Keywords:** Heterogeneity, Corruption, Collective Contests, Inequality, Selective Incentives

**JEL Classification:** D72, D73, D74

*Thanks to Sebastian Castro for research assistance.*

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## **A Tragic Solution to the Collective Action Problem: Implications for Corruption, Conflict and Inequality**

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Received: date / Accepted: date

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## 1 Introduction

It is shown in Nieva (2019) that a non productive enforcer colludes with a proper subset of workers and fights for a resource a la Tullock when he was expected in the grand coalition to adjudicate over property rights of the resource in such a way that all players get equal payoffs.<sup>1</sup> Such corrupt coalitions form if marginal productivity of labour is low enough as the equilibrium coalitions in the coalitional bargaining model he uses maximize per capita payoffs. More importantly, his results are consistent with the empirical correlation between corruption, conflict, inequality and productivity, where the first three variables are endogenous in his model.

However, strong assumptions are made: workers are homogeneous and there are no endogenous selective incentives; further, the prize, in the sense of the present paper, is a private good.

Recently, Nitzan and Ueda (2018) have shown that heterogeneous prize valuations in a competing coalition prevent effective use of such incentives which are meant to solve the collective-action problem (See Olson 1965): Effort is not provided efficiently (free-riding). Further, its winning probability goes down.

Hence, it is important to study this effectiveness in the presence of such an enforcer as for its implications, in particular, for corruption, conflict and inequality.<sup>2</sup>

In order to do that, we extend both Nieva (2019) and Nitzan and Ueda (2018). In the latter paper, several groups compete for a prize. Each member of a group has a different valuation for the mixed private-public-good prize. In the present paper, one player, the enforcer, is more effective at fighting than the other members in the group. All members in a group have identical cost functions. Before the contest takes place, the group agrees upon (or maybe there is a benevolent leader) a cost-sharing rule (which ranges from zero, no cost-sharing, to one, full cost-sharing) so that to maximize the sum of the expected utility of the group members .

We study the case where marginal effort cost exhibits constant elasticity.<sup>3</sup> If prize valuations are homogeneous, the first-best cost sharing rule is implemented, full sharing of the cost, if such an enforcer, that induces heterogeneity, is a member of the group; this is different than the result in Nieva (2019) and Nitzan and Ueda (2018); further, the group probability of winning goes up if such an enforcer is introduced following the former author as his analysis can be used with corner solutions.

If prize valuations are heterogeneous, more effectiveness of the enforcer leads still to partial cost-sharing found in Nitzan and Ueda (2018) whenever they only assume heterogeneity in prize valuations. However, the degree of

<sup>1</sup> Property rights are only well defined for output.

<sup>2</sup> In this paper, we do not focus on coalition formation, however, some conjectures could be made by the readers familiar with the paper in question.

<sup>3</sup> The associated cost function is also considered in Nitzan and Ueda (2018) who use a more general functional form; nevertheless, ours is popular in the literature,

cost-sharing goes up, and thus the probability of winning (in contrast to the latter authors) if the prize valuation of the enforcer is lower than the *Lehmer mean* of the other players' valuations. If higher, the degree of cost-sharing and such probability go down.

The intuition for these results is that a more effective enforcer induces workers to provide less effort relatively and this decreases the associated marginal cost of increasing effort of these members. In addition, group efficiency implies that if total effective group effort goes up in the coalition, the marginal increase of effort of such members goes down too. For the enforcer, we have opposite effects and these become stronger than the latter ones if the enforcer's valuation of the prize is higher than the Lehmer mean as he "overprovides effort"<sup>4</sup>. Thus, the marginal cost of increasing total effective group effort goes up in the latter case. The degree of cost sharing goes down as this induces a lower equilibrium total effective group effort.

The intuition if the prize valuations are homogeneous is that the enforcer's prize valuation is equal to the Lehmer mean and the degree of cost sharing does not change, actually, it is still full-cost sharing based on Nitzan and Ueda (2018).

"In the limit", with heterogeneity, when the effectiveness of the enforcer goes to infinity, the group chooses the first-best cost sharing rule unless the enforcer's prize valuation is too low. This happens as heterogeneity in valuations are meaningless relatively in terms of giving incentives to provide effort in the presence of such a mighty enforcer; also, efficiency is obtained because, *then*, the group leader's restricted maximization problem (where players choose effort simultaneously) coincide with the unrestricted one, where the leader chooses individual efforts, the one that yields group efficiency. If the prize valuation of the enforcer is low enough full sharing of the cost is obtained based on known properties of the Lehmer mean. As only then a corner solution occurs (corner solutions exist because the possible degrees of cost sharing are bounded), efficiency is not obtained. If the effectiveness of the enforcer increases when it is already very high, the group probability of winning goes down if the valuation of the enforcer is higher than the Lehmer mean, and goes up if the valuation of the enforcer is lower than the Lehmer mean. The intuition of the "non limiting case" can also be used in this case. The variety of these results contrasts with those in Nitzan and Ueda (2018).

Allowing for over cost-sharing, which is natural when the enforcer is not productive as in Nieva (2019), yields group efficiency even if the valuation of the enforcer is too low.

Following Nitzan and Ueda (2018), the first papers with heterogeneity in collective contests can be found in Baik (2008), Esteban and Ray (2011), Epstein and Mealem (2009), Ryvkin (2011), and Nitzan and Ueda (2013). Most of these papers focus on the effect of heterogeneity in group effort. With respect to considering selective incentives with imperfect information, Nitzan

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<sup>4</sup> Actually, a more simple but accurate explanation is very complex to formulate based on our formal results.

and Ueda (2011) are the first ones to allow for endogenous prize-sharing rules considering homogeneous prize valuations which end up maximizing the utilitarian group-welfare as conjectured by Olson. Nitzan and Ueda (2018) introduce heterogeneity to account for the persistence of collective action problems even with endogenous incentives; further, they show that the group probability of winning goes down in the presence of heterogeneity and endogenous incentives.

As for Nieva (2019), who finds that the introduction of an enforcer increases the probability of winning of the group contesting over the property right of a "private good" (more precisely the contest is over a piece of land, and output using such a resource is private), the present paper with endogenous incentives contributes to the theoretical literature on corruption as the results imply that, nevertheless, the probability of winning of such groups goes up if the prize valuation of the enforcer is not too high. Further, in the limit, if the prize valuation of the enforcer is not too low, these coalitions are even group efficient, and thus, the collective action problem is solved; as this corresponds to low marginal product of labour in the paper in question, tragically, corrupt coalitions in developing countries are even at their best when being corrupt; "on the other side of the moon", with publicness, or if productivity is high enough, non corrupt coalitions are at their best in the presence of such an enforcer.

As an increase in the winning probability drives the results on coalition formation in Nieva (2019), our results might strenghten these ones (If we would allow for coalition formation). On the other hand, if the enforcer prize valuation is higher than the Lehmer mean, that is if it has more stake, then this would imply less corruption in low productivity countries. In a sense, this is kind of obvious, but, in any case, it is not consistent with the negative correlation between corruption, productivity, conflict and inequality as discussed in the paper in question. It is important to emphasize that even if the stake of the enforcer is too low, the corrupt coalition is not efficient anymore, but still its probability of winning goes up if the enforcer gets very effective. Even more tragic, over cost-sharing solves the collective action problem in the latter case.

In the second section, we present the model. In the third section, we discuss the first-best cost sharing rule and characterize the equilibrium cost-sharing rules. In the fourth section, we conclude.

## 2 The Model

There are  $m$  groups<sup>5</sup> that compete a la Tullock over a mixed private-public-good prize. Let  $N_i$  be the number of members in group  $i$ . For notation purposes and comparisons with Nitzan and Ueda (2018), we will assume that all groups have an enforcer, player 1, who is more effective at fighting than the other ones in the group by a factor of  $z > 1$ , the *effectiveness* of effort; the other

<sup>5</sup> We follow the notation and structure of the paper in Nitzan and Ueda (2018), rather than that in Nieva (2019).

ones are identical in this respect. All members of the group  $i$ , for all groups, choose simultaneously effort  $a_{ik}$ ,  $k \in N_i$ , to enhance the probability of winning of the group. The *total effective* effort in group  $i$  is  $A_i = \left( \sum_{k=2}^{N_1} a_{ik} \right) + za_{i1}$ . The probability of winning for group  $i$  is  $\frac{A_i}{A}$ , where  $A = \sum_{j=1}^m A_j$  is the total amount of effective effort of all coalitions.

Before the contest takes place, the group agrees upon over a cost sharing rule, the incentive device (the actions in the first stage could be thought of as benevolent group leaders choosing simultaneously such a cost-sharing rule). The objective is to maximize the sum of utilities of members of the group assumed to be risk neutral. The total valuation of the prize is  $V_i = \sum_{k=1}^{N_i} v_{ik}$ , where  $v_{ik} > 0$  is the valuation of the prize of player  $k$  in group  $i$ . The distribution of stakes is  $v_i = (v_{i1}, \dots, v_{iN_i})$ . The effort cost function is identical for all members and thrice differentiable. Additionally, we have that  $c_i(0) = 0$ ,  $c'_i(a) > 0$  and  $c''_i(a) > 0$  for all  $a > 0$ . Also,  $\lim_{a \rightarrow 0} c'_i(a) = 0$ .<sup>6</sup>

With respect to selective incentives, a fraction  $\delta$  of total group cost is shared within the group equally. Formally, the cost of individual  $k$  in group  $i$  is

$$(1 - \delta_i)c_i(a_{ik}) + \delta_i \frac{\sum_{p=1}^{N_1} c_i(a_{ip})}{N_i}.$$

A value of one for  $\delta$  stands for the highest degree of cost sharing, or that cost is fully shared. A value of zero refers to the standard case. The utility of individual  $k$  in group  $i$  is then

$$EU_{ik} = \frac{A_i}{A} v_{ik} - \left\{ (1 - \delta_i)c_i(a_{ik}) + \delta_i \frac{\sum_{p=1}^{N_1} c_i(a_{ip})}{N_i} \right\}. \quad (1)$$

In the second stage, the member  $k$  of group  $i$  chooses  $a_{ik}$  to maximize (1) given  $\delta_i$ . In the first stage, the social planner chooses  $\delta_i$  to maximize the sum of utilities of the members of the group  $\sum_{k=1}^{N_i} EU_{ik}$  anticipating  $EU_{ik}$  for each  $k$ .

We assume, as in Nitzan and Ueda (2018), that cost sharing rules chosen in the other groups are not observable for members of a given group (for a discussion on this assumption, see that paper). As a solution concept, we use perfect Bayesian equilibrium. Denote the beliefs of player  $k$  in group  $i$  by  $\mu_{ik}(\delta_i)$ . This is a probability distribution over  $[0, 1]^{m-1}$ . This is the space of cost-sharing rules chosen by the other groups with typical element  $\delta_{-i} = (\delta_1, \dots, \delta_{i-1}, \delta_{i+1}, \dots, \delta_m) \in [0, 1]^{m-1}$ . We allow only for pure strategies.

Suppose  $\delta^*$  is an equilibrium profile of cost-sharing rules. Along the equilibrium path, the beliefs of player  $k$  in  $i$  are such that she expects the other leaders to play  $\delta_{-i}^*$ , that is,  $\mu_{ik}(\delta_{-i}^*/\delta_i^*) = 1$ . If the leader of group  $i$  deviates it is assumed that the same is believed appealing to the “no-signaling-what-you-don’t-know” condition as in Fudenberg and Tirole (1991). What follows is the

<sup>6</sup> This latter assumption ensures uniqueness and simplicity. See Nitzan and Ueda (2013) for a different assumption that allows the possibility of non-contributors.

characterization of equilibrium choices by members of each group. Lemma 1 in Nitzan and Ueda (2018) changes as follows:

**Lemma 1** *The equilibrium effort level by player  $k$  of group  $i$  who knows the cost-sharing rule  $\delta_{-i}^*$  (i.e. at the information set indexed by  $\delta_{-i}^*$ ) is described by a strictly increasing differentiable function  $a_{ik}^*(\delta_{-i}^*)$  defined by the following equation*

$$\frac{\sum_{j \neq i} A_j^*(\delta_j^*)}{\left\{ \sum_{j \neq i} A_j^*(\delta_j^*) + A_i^*(\delta_i) \right\}^2} z_k v_{ik} - \left\{ (1 - \delta_i) + \frac{\delta_i}{N_i} \right\} c'_i(a_{ik}^*(\delta_i)) = 0, \quad k = 1, \dots, N_i, \quad (2)$$

where

$$A_i^*(\delta_i) = \sum_{k=i}^{N_i} z_k a_{ik}^*(\delta_i), \quad (3)$$

given the other groups equilibrium cost-sharing rule  $\delta_j^*$ ,  $j \neq i$  and  $z_k$  is such

that  $z_1 = z$ ,  $z_k = 1$  for  $k \neq 1$  if the group contains an enforcer. In Nitzan and Ueda  $z_k = 1$  for any  $k, i$ .

*Proof* Equation (2) is the first order necessary condition of the maximization problem implied by (1). As for strict convexity of the cost function, the solution to the maximization problem for each player  $k \in N_i$  is unique and positive.

As  $\sum_{j \neq i} A_j^*(\delta_j^*) = A - A_i$ , we can write (2) as

$$\frac{A - A_i}{A^2} z_k v_{ik} = \left\{ (1 - \delta_i) + \frac{\delta_i}{N_i} \right\} c'_i(a_{ik}^*(\delta_i)) = 0, \quad k = 1, \dots, N_i.$$

As  $c'_i(a)$  is strictly increasing, the inverse exists, hence; we can write

$$\rho_{ik}(A, A_i, v_{ik}, \delta_i) = (c'_i)^{-1} \left[ \frac{1}{(1 - \delta_i) + \frac{\delta_i}{N_i}} \frac{A - A_i}{A^2} z_k v_{ik} \right] = a_{ik}.$$

In the literature, this is called the replacement function, as in Cornes and Hartley (2003), and, thus,  $A$  and  $A_i$  are independent in these individual functions. The function  $\rho_{ik}$  is continuous and decreasing in  $A_i$  in the relevant range. As  $A_i$  tends to  $A$ ,  $\rho_{ik}$  tends to zero. As  $A_i \rightarrow 0$ , it tends to a finite number.

It follows that the summation of  $\rho_{ik}$ 's have the same properties. Thus, there exists a unique value of  $A_i^*$  such that  $A_i^* = \sum_{k=1}^{N_i} \rho_{ik}(A, A_i, v_{ik}, \delta_i)$  and  $0 < A_i^* < A$ . Thus, we can define  $A_i(A, v_i, \delta_i)$  as an implicit function. Clearly,  $A_i(\cdot)$  is strictly increasing in  $\delta_i$ .

After using (2), for the enforcer and any other player  $k \neq 1$ , we obtain after dividing the two equations

$$\frac{1}{v_{ik}} c'_i(a_{ik}(A_i; v_i)) = \frac{1}{z v_{i1}} c'_i(a_{i1}(A_i; v_i)). \quad (4)$$

As  $c'_i(a)$  is strictly increasing and, after using equation (3), the claim follows  $\square$



Based on the previous Lemma, the planner in the group can control individual efforts in the group by choosing the degree of costs sharing  $\delta$  given other groups efforts  $A_k$ ,  $k \neq i$ . This will imply that he basically chooses  $A_i$  from an interval as characterized in the next lemma. The proof follows identically to Lemma 2(a) in Nitzan and Ueda (2018).

**Lemma 2** *For each level of efforts by the other groups  $\sum_{j \neq i} A_j$ , group  $i$  can attain the aggregate group effort  $A_i$  if and only if it belongs to the closed interval  $\left[ A_i^L \left( \sum_{j \neq i} A_j \right), A_i^H \left( \sum_{j \neq i} A_j \right) \right]$ , where the endpoints are uniquely given by the equations*

$$\sum_{k=1}^{N_i} (c'_i)^{-1} \left[ \frac{\sum_{j \neq i} A_j}{\left( \sum_{j \neq i} A_j + A_i^L \left( \sum_{j \neq i} A_j \right) \right)^2} z_k v_{ik} \right] = A_i^L \left( \sum_{j \neq i} A_j \right), \text{ and}$$

$$\sum_{k=1}^{N_i} (c'_i)^{-1} \left[ N_i \frac{\sum_{j \neq i} A_j}{\left( \sum_{j \neq i} A_j + A_i^H \left( \sum_{j \neq i} A_j \right) \right)^2} z_k v_{ik} \right] = A_i^H \left( \sum_{j \neq i} A_j \right).$$

As we will be interested in the optimal group level of effort, we define the cost of inducing  $A_i$  as

$$E_i(A_i; v_i) = \sum_{k=1}^{N_i} c_i(a_{ik}(A_i; v_i))$$

Nitzan and Ueda (2018) call this the distorted group cost of  $i$  as the planner cannot choose individual efforts.

Lemma 3 in Nitzan and Ueda (2018) is reformulated as follows, where the implicit derivation uses equation (3) and (4). Their proof still holds if  $z > 1$ .

**Lemma 3** *The equation*

$$\frac{\partial}{\partial A_i} E_i(A_i; v_i) = \sum_{k=1}^{N_i} c'_i(a_{ik}(A_i; v_i)) \frac{\frac{z_k v_{ik}}{c''_i(a_{ik}(A_i; v_i))}}{\sum_{p=1}^{N_i} \frac{z_p^2 v_{ip}}{c''_i(a_{ip}(A_i; v_i))}} \quad (5)$$

*holds for  $A_i > 0$ . Furthermore,  $\lim_{A_i \rightarrow 0} \frac{\partial}{\partial A_i} E_i(A_i; v_i) = 0$ .*

Based on our assumptions, the payoff of the group planner as a function of the distorted cost function is:

$$\frac{A_i}{\sum_{j=1}^m A_j} V_i - E_i(A_i; v_i) \quad (6)$$

Then the model is simplified by allowing group planners to choose group efforts simultaneously. Formally, an equilibrium of this reduced contest is a

profile of group efforts  $A_j^*$ ,  $j = 1, \dots, m$  such that  $A_i^*$  solves

$$\max_{A_i > 0} \frac{A_i}{\sum_{j \neq i} A_j^* + A_i} V_i - E_i(A_i; v_i) \quad (7)$$

$$\text{subject to } A_i^L \left( \sum_{j \neq i} A_j^* \right) \leq A_i \leq A_i^H \left( \sum_{j \neq i} A_j^* \right)$$

for all  $i = 1, \dots, m$ .

It is not hard to see that if an equilibrium of this reduced model of contest uniquely exists, the same is true for pure-strategy perfect Bayesian equilibrium of the original collective contest model with cost sharing.

Nitzan and Ueda (2018) prove that the equilibrium in this model is unique if the distorted group cost  $E_i(A_i; v_i)$  is convex in  $A_i$  (See proposition 1 in Nitzan and Ueda (2018)). Thus, to prove existence and uniqueness, we show that it is convex for the case of constant elasticity of marginal effort cost whenever there is an enforcer.

**Lemma 4** *The distorted group cost  $E_i(A_i; v_i)$  is convex in  $A_i$  for the constant elasticity case.*

*Proof* From equation (4), for  $k \neq 1$

$$v_{ik} = \frac{c'_i(a_{ik}(A_i; v_i))}{c'_i(a_{i1}(A_i; v_i))} v_{i1} z. \quad (8)$$

Replace  $v_{ik}$  using the latter expression in (5) in the numerator and denominator for each  $k$  in the right hand side expression. Factor out  $\frac{v_{i1} z}{c'(a_{i1})}$  and cancel out to obtain

$$\frac{\partial E_i(A_i; v_i)}{\partial A_i} = \sum_{k=1}^{N_i} c'_i(a_{ik}(A_i; v_i)) \frac{\frac{c'_i(a_{ik}(A_i; v_i))}{c'_i(a_{ik}(A_i; v_i))}}{\sum_{p=1}^{N_i} \frac{z_p c'_i(a_{ip}(A_i; v_i))}{c''_i(a_{ip}(A_i; v_i))}} \quad (9)$$

As  $c_i(a) = K_i \frac{a^{1+\alpha_i}}{1+\alpha_i}$ , (9) becomes  $\frac{\partial E_i(A_i; v_i)}{\partial A_i} = \sum_{k=1}^{N_i} K_i a^{\alpha_i} \frac{a_{ik}}{\sum_{p=1}^{N_i} z_p a_{ip}}$ . After using the product rule to obtain the  $2^{\text{nd}}$  partial derivative of  $E_i(A_i; v_i)$  with respect to  $A_i$ , one of the two  $k$ -terms in this derivative for given  $k$  is

$$K_i a^{\alpha_i} \left[ \frac{\frac{a_{ik}}{\sum_{p=1}^{N_i} z_p a_{ip}}}{\sum_{p=1}^{N_i} z_p a_{ip}} - \frac{a_{ik} \sum_{p=1}^{N_i} \frac{z_p a_{ip}}{\sum_{t=1}^{N_i} z_t a_{it}}}{\left( \sum_{p=1}^{N_i} z_p a_{ip} \right)^2} \right].$$

Clearly, it vanishes as  $\sum_{p=1}^{N_i} \frac{z_p a_{ip}}{\sum_{t=1}^{N_i} z_t a_{it}} = 1$ , and, hence, the distorted group cost is convex as the other term in the second partial derivative is positive  $\square$

### 3 Efficiency Analysis and Equilibrium Cost-Sharing Rules

#### 3.1 Contests by Fully Regulated Groups

Before we characterize our results, we study the situation where the group leader can achieve efficiency by controlling the level of individual efforts in the group. The planner maximizes, given  $A_j$  for  $j \neq i$ , the following expression:

$$\frac{\sum_{k=1}^{N_i} z_k a_{ik}}{\sum_{j \neq i} A_j + \sum_{k=1}^{N_i} z_k a_{ik}} V_i - \sum_{k=1}^{N_i} c_i(a_{ik}). \quad (10)$$

where  $V = \sum_{k=1}^{N_i} v_{ik}$ . First order conditions are

$$z_k \frac{A - A_i}{A^2} V_i - c'_i(a_{ik}) = 0, \quad k = 1, \dots, N_i.$$

Efficiency, thus, implies that

$$c'_i(a_{ik}) = \frac{z_k}{z_1} c'_i(a_{i1}), \quad k \neq 1 \quad (11)$$

Thus, an equivalent way of formulating the group planner's problem is

$$\max_{A_i} \frac{A_i}{A} V_i - \sum_{k=1}^{N_i} c_i(a_{ik}) \quad (12)$$

where  $a_{ik}$  is defined implicitly by (11) and (3).

#### 3.2 Cost-Sharing Rules and Efficiency Analysis with Homogeneous Valuations

We can get the analogous of proposition 2 in Nitzan and Ueda (2018) but including an enforcer.

**Proposition 1** *In the presence of an enforcer, provided we have constant elasticity of marginal effort cost, when all individuals have identical prize valuations, the cost is fully shared in equilibrium. This cost-sharing rule is first-best.*

We will prove this in a more general set up, that is, considering heterogeneity in prize valuations. Nevertheless, it is instructive to give the argument in an alternative way. Set  $\delta = 1$  in (2), sum over all players in  $N_i$  and observe that this is equivalent to the summation of the  $N_i$  first order conditions of the social planner's problem in (10).

Thus, in contrast with Nitzan and Ueda (2018), our source of heterogeneity does not prevent endogenous cost sharing to lead to group efficiency.

#### 3.3 Cost-Sharing Rules and Efficiency Analysis with Heterogeneity

Let

$$\gamma_i = 1 - \delta_i + \frac{\delta_i}{N_i}, \quad (13)$$

thus, if  $\delta_i = 0$ ,  $\gamma_i = 1$  and if  $\delta_i = 1$ ,  $\gamma_i = \frac{1}{N_i}$ . As the distorted group cost function  $E_i(A_i)$  is convex and, hence, the equilibrium in our game exists and is unique, we can use the first order conditions of the group leader in (7). The analogous result to that in Nitzan and Ueda (2018) (See Proposition 4) is

**Proposition 2** *In equilibrium, in the presence of an enforcer, for the case of constant elasticity of marginal effort cost, the cost sharing rule chosen by group  $i$  satisfies the inequalities*

$$\gamma_i \leq (\geq) \frac{\sum_{k=1}^{N_i} (z_k v_{ik})^{\frac{1}{\alpha_i} + 1}}{V_i \sum_{k=1}^{N_i} z_k^{\frac{1}{\alpha_i} + 1} v_{ik}^{\frac{1}{\alpha_i}}}, \quad (14)$$

if  $\gamma_i > \frac{1}{N_i}$  ( $\gamma_i < 1$ ).

*Proof* From the first order conditions of (7), we get

$$\frac{\sum_{j \neq i} A_j}{\left(\sum_{j=1}^m A_j\right)^2} \sum_{k=1}^{N_i} v_{ik} - \sum_{k=1}^{N_i} c'_i(a_{ik}(A_i; v_i)) \frac{\frac{z_k v_{ik}}{c'_i(a_{ik}(A_i; v_i))}}{\sum_{p=1}^{N_i} \frac{z_k^2 v_{ip}}{c'_i(a_{ip}(A_i; v_i))}} \leq (\geq) 0,$$

if  $\delta_i < 1$  ( $\delta_i > 0$ ).

But each individual in  $N_i$  is solving his maximization problem according to (2). After using instead  $\gamma_i$ , from (2), we get

$$\frac{\sum_{j \neq i} A_j}{\left(\sum_{j=1}^m A_j\right)^2} \frac{z_k v_{ik}}{\gamma_i} = c'_i(a_{ik}(A_i; v_i)),$$

for all  $k = 1, \dots, N_i$ .

After combining the last two expressions, we get

$$\gamma_i \leq (\geq) \frac{\sum_{k=1}^{N_i} c'_i(a_{ik}(A_i; v_i)) \frac{\frac{z_k v_{ik}}{c'_i(a_{ik}(A_i; v_i))}}{\sum_{p=1}^{N_i} \frac{z_k^2 v_{ip}}{c'_i(a_{ip}(A_i; v_i))}}}{\sum_{k=1}^{N_i} \frac{c'_i(a_{ik}(A_i; v_i))}{z_k}} = \frac{\frac{\partial E_i}{\partial A_i}(A_i^*; v_i)}{\sum_{k=1}^{N_i} \frac{c'_i(a_{ik}(A_i^*; v_i))}{z_k}}, \quad (15)$$

if  $\gamma_i > \frac{1}{N_i}$  ( $\gamma_i < 1$ ), where the equality follows from (8) which was used to derive (9).

Finally, after letting  $c_i(a) = K_i \frac{a^{1+\alpha_i}}{1+\alpha_i}$ , we obtain after some algebra<sup>7</sup> (14)  $\square$

For this cost function, if  $z = 1$ , Nitzan and Ueda (2018) have shown that the solution is interior, that is,  $\frac{1}{N_i} < \gamma_i < 1$  (See Proposition 5a) if there is heterogeneity in prize valuations. Further, with identical agents and no enforcer, they confirm that there is full cost sharing. It is easy to see that, in

<sup>7</sup> More details and discussion of this derivation can be found later on.

our case, with no heterogeneity in valuations, the same is true if in addition  $z > 1$ , as then  $\gamma_i = \frac{1}{N_i}$  (see their proof of proposition 5(b) where it is shown that if  $\gamma_i \leq 1$ , there is full cost sharing, otherwise, partial cost sharing).

Thus with  $z$  slightly greater than 1 and heterogeneity the solution is still interior and the following proposition follows

**Proposition 3** *If  $z$  is slightly greater than 1 and we have constant elasticity of marginal effort cost, the degree of cost-sharing goes up if the prize valuation of the enforcer is lower than the Lehmer mean of the other players' valuations, if it is higher, it goes down.*

*Proof* As the result is interior, the result follows after taking the derivative of the expression in (14) with respect to  $z$ , where the Lehmer mean for the other players is  $\frac{\sum_{k=2}^{N_i} v_{ik}^{\frac{1}{\alpha_i}+1}}{\sum_{k=2}^{N_i} v_{ik}^{\frac{1}{\alpha_i}}}$   $\square$

The derivation of the economic intuition based on the mathematics of the proof is complex.

First, note that the equation in (15) can be expressed as the equation of marginal revenue equal to marginal cost

$$\gamma_i \sum_{k=1}^{N_i} \frac{c'_i(a_{ik}^*)}{z_k} = \frac{\partial E_i}{\partial A_i}(A_i^*; v_i) \quad (16)$$

All terms in the summation in the right hand side or left hand side are terms that are different because of heterogeneity not just in the valuation of the prize but in fighting effectiveness given by  $z > 1$ . Thus, what matters in

$$c'_i(a_{ik}) = \frac{K_i z_k v_{ik} A_i^{\alpha_i}}{\left( \sum_{k=1}^{N_i} z_k^{\frac{1}{\alpha_i}+1} v_{ik}^{\frac{1}{\alpha_i}} \right)^{\alpha_i}}$$

is only  $z_k v_{ik}$ . The latter expression follows after using  $c'_i(a_{ik}) = K_i a_{ik}^{\alpha_i}$ , where

$$a_{ik} = \frac{(z_k v_{ik})^{\frac{1}{\alpha_i}}}{\sum_{k=1}^{N_i} z_k^{\frac{1}{\alpha_i}+1} v_{ik}^{\frac{1}{\alpha_i}}} A_i$$

which is in turn obtained after using (3) and (4),

After using similar arguments, in other components of summation terms like  $\frac{\partial a_{ik}}{\partial A_i}$  in  $c'_i(a_{ik}) \frac{\partial a_{ik}}{\partial A_i}$  in (9), (16) can be expressed as follows after canceling out terms:

$$\gamma_i \sum_{k=1}^{N_i} v_{ik} = \sum_{k=1}^{N_i} z_k v_{ik} \frac{(z_k v_{ik})^{\frac{1}{\alpha_i}}}{\sum_p z_p^{\frac{1}{\alpha_i}+1} v_{ip}^{\frac{1}{\alpha_i}}} = \frac{\left( \sum_{k=2}^{N_i} v_{ik}^{\frac{1}{\alpha_i}+1} \right) + (z v_{i1})^{\frac{1}{\alpha_i}+1}}{\left( \sum_{p=2}^{N_i} v_{ik}^{\frac{1}{\alpha_i}} \right) + z^{\frac{1}{\alpha_i}+1} v_{i1}^{\frac{1}{\alpha_i}}}$$

Note that  $z_k v_{ik}$  in the middle term is the term that corresponds to  $c'_i(a_{ik})$  after terms have been canceled out. It is easy to see, and, actually, it can be shown, that when  $z$  goes up the last term in the numerator in the last expression of the equality,  $(z v_{i1})^{\frac{1}{\alpha_i}+1}$ , has a higher effect than  $z^{\frac{1}{\alpha_i}+1} v_{ik}^{\frac{1}{\alpha_i}}$  in the denominator, as  $v_{i1}^{\frac{1}{\alpha_i}+1} > v_{i1}^{\frac{1}{\alpha_i}}$  provided  $v_{i1}$  is high enough (more precisely higher than the Lehmer mean of the other players). Note that this occurs because when  $A_i$  goes up there are two effects that affect  $c'_i(a_{ik})$  and  $\frac{\partial a_{ik}}{\partial A_i}$  respectively in each  $k$ -summation term in  $\frac{\partial E_i}{\partial A_i}(A_i^*; v_i)$ . If  $v_{i1}$  is high enough the marginal cost is higher for the enforcer as "he provides more effort relatively". In addition, his marginal increase in effort,  $\frac{\partial a_{i1}}{\partial A_i}$ , is higher due the enforcer's higher valuation of the prize. As this two effects together imply  $z^{\frac{1}{\alpha_i}+1} v_{ik}^{\frac{1}{\alpha_i}+1}$  in the numerator in the summation term and we have only  $z^{\frac{1}{\alpha_i}+1} v_{ik}^{\frac{1}{\alpha_i}}$  in the denominator, the marginal increase of the ratio goes up if  $v_{i1}$  is high enough. This effect eventually offsets the decreasing effect of  $z$  going up on the marginal cost of increasing  $A_i$  for the other players, and, thus, the results.

With respect to the previous proposition's implications for the group's probability of winning we have a "standard" result as the solution is interior.

**Proposition 4** *If  $z$  is slightly greater than 1 and we have constant elasticity of marginal effort cost, the winning probability of the group goes up if the prize valuation of the enforcer is lower than the Lehmer mean of the other players' valuations; if it is higher, it goes down.*

We can use the share function approach as in Cornes and Hartley (2003) but adapted to collective contests with endogenous selective incentives as in Nitzan and Ueda (2018). The latter authors show that we can define instead the probability of winning of group  $i$ ,  $\pi_i = \frac{A_i}{A}$ , for each  $i = 1, \dots, m$ , which is strictly decreasing with respect to total effective effort  $A$ . Further it tends to 1 as  $A \rightarrow 0$  and it tends to zero when  $A \rightarrow \infty$ . Thus, the sum of  $\pi_i$  over all  $i$  is also strictly decreasing in  $A$ , and instead tends to  $m$  and zero respectively. Hence, in the first case, there exists a unique  $A$ , the equilibrium total effective effort, such that the sum equals one. Clearly, if  $z$  goes up in  $A_i$ ,  $\pi_i$  goes up for given  $A$  (recall that if the degree of cost-sharing goes up  $A_i$  goes up, and as for the previous proposition if  $z$  goes up this degree goes up). Then, the new equilibrium  $A$  has to go up, but then  $\pi_j$ ,  $j \neq i$ , in other groups have to go down and the result follows.

**Proposition 5** *If  $z \rightarrow \infty$ ,  $\gamma_i \rightarrow \frac{v_{i1}}{V}$ . Further we have*

a) *If  $v_{i1} > \frac{\sum_{k=2}^{N_i} v_{ik}}{N_i - 1}$  then for very high effectiveness the equilibrium exhibits partial cost-sharing.*

b) *If  $v_{i1} < \frac{\sum_{k=2}^{N_i} v_{ik}}{N_i - 1}$  and  $v_{i1} < \frac{\sum_{k=2}^{N_i} v_{ik}^{\frac{1}{\alpha_i}+1}}{\sum_{k=2}^{N_i} v_{ik}^{\frac{1}{\alpha_i}}}$  then for very high effectiveness the equilibrium exhibits full cost-sharing.*

*Proof* The first part follows by taking the limits of the equality version of (14). For the second part, note that if  $\alpha_i \rightarrow \infty$ , the Lehmer mean tends to  $\frac{\sum_{k=2}^{N_i} v_{ik}}{N_i-1}$ , the arithmetic mean of the players but for the enforcer. Thus, if a) holds, in the limit,  $\gamma_i \rightarrow \frac{v_{i1}}{V} > \frac{1}{N_i}$ . As for the proof in proposition 5(a) in Nitzan and Ueda (2018), the claim follows. Analogously, if b) holds  $\gamma_i \rightarrow \frac{v_{i1}}{V} < \frac{1}{N_i}$ , and the claim follows as for the proof in proposition 5(b) in the paper in question  $\square$

Note that if a) holds, the solution gets close to the first best cost-sharing rule. This occurs as, first, in the limit all players but the enforcer don't provide any effort (this is consistent with the limit version of (4)). Second, if effectiveness is very high, solutions are interior and the first order conditions get closer to the ones in the group planner's unrestricted problem in (12) or, equivalently, (10) (See last proposition for a formal result in the context of over cost-sharing). If b) holds, the level of group effort  $A_i$  that solves (12) is strictly higher than  $A_i^H \left( \sum_{j \neq i} A_j \right)$  for high effectiveness of the enforcer  $z$  as we have a corner solution in (7) and, thus, it is not first best.

With respect to the winning probability of groups with an enforcer, we have

**Proposition 6** *The probability of the group with the enforcer winning increases as  $z$  goes up for  $z$  very high if the valuation of the prize of the enforcer is lower than the Lehmer mean of the other players' valuation.*

*Proof* There are two cases. The first one corresponds to the situation where  $\gamma_i \rightarrow \frac{v_{i1}}{V} > \frac{1}{N_i}$ . If  $z$  goes up when  $z$  is high enough then we can apply the previous proposition as the derivative is evaluated at an interior solution. The second one corresponds to the situation where  $\gamma_i \rightarrow \frac{v_{i1}}{V} < \frac{1}{N_i}$ . If  $z$  goes up when  $z$  is high enough and the corresponding equilibrium implies  $\gamma_i < \frac{1}{N_i}$  then we can set  $\delta = 1$  in (2) and use the standard share function approach but with an enforcer with no selective incentives as in Nieva (2019) to evaluate the corresponding derivative  $\square$

Thus the collective action problem is solved when the enforcer's prize valuation is not too low.

### 3.4 Over Cost-Sharing Rules and Efficiency Analysis with Heterogeneity

In the last subsection, we found that "in the limit", when the effectiveness of the enforcer goes to infinity, the first best cost-sharing rule is chosen provided the valuation of the enforcer is not too low. In the latter case, formally speaking, we have a corner solution. If the condition (b) in the Proposition 5 holds, the level of group effort  $A_i$  that solves (12), the unrestricted (or group efficient) first order condition of the leader's problem, is strictly higher than the group's effort in the restricted leader's problem  $A_i^H \left( \sum_{j \neq i} A_j \right)$  for high

effectiveness of the enforcer  $z$  and, thus, the latter group effort level is not first best.

However, let us allow for over cost-sharing, which is natural in a corruption environment as the enforcer in Nieva (2019) is not productive and only derives income from transfers. We then have our last proposition:

**Proposition 7** *With over cost-sharing and heterogeneity, if the effectiveness of the enforcer goes to infinity, then the cost-sharing rule chosen tends to the first-best one.*

*Proof* It only suffices to explain the case (b) in Proposition 5. As long as  $\delta < \frac{N_i}{N_i-1}$  each player's maximization problem in group  $i$  is well defined, that is, a maximum exists and, further, it is unique. After solving for  $\delta$  in (13), we get  $\delta = \frac{N_i(1-\gamma_i)}{N_i-1}$ . Note that the sharing rule chosen in the restricted leader's problem  $\gamma_i \rightarrow \frac{v_{i1}}{V}$  as  $z \rightarrow \infty$ . Thus, as  $0 < \frac{v_{i1}}{V} < 1$ , the maximization problem in question is well defined.

The last part of the proof consists of showing that in the limit the first order conditions associated to the leader's unrestricted problem, (12), are equivalent to those in the restricted one, (7). The easiest way to proceed is to take the partial derivative with respect to  $a_{ik}$  for  $k \neq 1$  in the unrestricted problem in (10). Then, use  $c'_i(a_{ik}) = K_i a_{ik}^{\alpha_i}$  where, after using (11) and (3),

$$a_{ik} = \frac{1}{N_i - 1 + z_k^{\frac{1}{\alpha_i} + 1}} A_i.$$

Thus, we obtain the following equation that defines implicitly a function  $A_i(z)$ :

$$\frac{A - A_i}{A} = \frac{K_i A_i^{\alpha_i}}{V_i} \frac{1}{\left(N_i - 1 + z_k^{\frac{1}{\alpha_i} + 1}\right)^{\alpha_i}}. \quad (17)$$

Now for the restricted case, use in the first order condition for a member  $k \neq 1$  of group  $i$ , (3), and the equilibrium value of  $\gamma$  using the equality version of (14). After some cancelling out and taking L'Hôpital's rule, we obtain

$$\frac{A - A_i}{A} = \frac{K_i A_i^{\alpha_i}}{V_i} \frac{v_{i1}}{\left(\left(\sum_{p=2}^{N_i} v_{ik}^{\frac{1}{\alpha_i}}\right) + z^{\frac{1}{\alpha_i} + 1} v_{i1}^{\frac{1}{\alpha_i}}\right)^{\alpha_i}}. \quad (18)$$

As effectiveness goes to infinity marginal costs in both (17) and (18) converge to zero and, hence, our claim follows as the associated equilibrium limiting equilibrium value  $\gamma > \frac{v_{i1}}{V}$ , and so all maximization problems are well defined  $\square$

## 4 Conclusion

We have introduced an enforcer that is more effective at fighting in a collective contest where groups choose a cost sharing rule. Members choose, given



the sharing rule, effort simultaneously so that the group fights over a mixed private-public good prize that is valued differently.

In contrast to the previous literature, our source of heterogeneity solves the collective action problem if valuations are homogeneous. With heterogeneity it solves it "in the limit" if the valuation of the enforcer is not too low. The intuition is that more effectiveness of the enforcer leads him to specialize relatively in fighting. If his stake is not too high he does not over provide effort and hence it is less costly to increase group effort. The result is tragic as if the prize is private, which corresponds to low enough marginal productivity of labour in terms of Nieva (2019), a corrupt coalition is an equilibrium outcome and so are conflict and inequality. Further, the corrupt coalition solves its collective action problem provided the stake of the enforcer is not too low; on the other hand, if productivity is high enough (the prize is more public) non corrupt coalitions solve its collective action problem too. But, even if the valuation of the enforcer is low enough, the probability of winning goes up as the enforcer's effectiveness goes up. Tragically, allowing for over cost-sharing yields group efficiency in the latter case.

Based on Nitzan and Ueda (2018), we conjecture that the implications on corruption, conflict and inequality would not change if considering more general cost functions as in that paper.<sup>8</sup> Also it is worthwhile to check if prize sharing rules leads to efficiency if the stake is low enough for other cases where the enforcer may be productive and over cost-sharing does not seem reasonable as the latter authors point out.

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<sup>8</sup> Our cost functions are a particular case of the more general ones where the relative rate of change of the marginal effort cost is decreasing. If the relative rate of change of the marginal effort costs is increasing the authors show that a corner solution results. Increasing the effectiveness of the enforcer should yield lower marginal costs or higher, depending on the enforcer's valuation, and the same associated implications as in this present study.

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